CP violation in supersymmetric models

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For supersymmetric extensions of the standard model, we obtain necessary conditions for CP invariance expressed in terms of weak-basis invariants. Among other applications, these conditions allow a simple identification of independent sources of CP violation and a simple exact count of CP-violating phases for any choice of couplings and masses.

Within the minimal standard model with three generations, CP violation[1] arises from the presence of a physical complex phase in the Kobayashi-Maskawa (KM) matrix.[2] Many other sources of CP violation can arise in extensions to the standard model (see, for example, Ref. 3). Investigation of CP violation could therefore provide evidence for new physics. Of special interest are supersymmetric gauge theories, which may provide a framework for a solution to the gauge hierarchy problem[4] and may enable the incorporation of gravity into unified models, for example, via superstrings. Indeed, in the low-energy limit, certain $N=1$ supergravity models can provide supersymmetric extensions of the standard model (see, for example, Ref. 5). If the set of new fields required by supersymmetry is minimal, these extensions differ only by the form of soft-supersymmetry-breaking operators appearing and are generically referred to as the supersymmetric standard model. In the absence of direct experimental evidence for superpartners, the low-energy properties of the supersymmetric standard model are important for its phenomenological viability. Investigation of its CP content is therefore of interest.

In this paper, we present a procedure for the characterization of CP invariance at the Lagrangian level[6–25] in the supersymmetric standard model and its nonminimal extensions. A related approach has been introduced and applied in the context of the standard model in Refs. 26–31. The key idea is to allow for the most general CP transformation leaving the Lagrangian invariant. One then obtains a series of relations that must be satisfied by the various interaction terms of the Lagrangian for CP invariance to hold. Straightforward manipulation of these relations yields simple necessary conditions for CP invariance. These conditions are expressed in terms of quantities that are weak-basis independent; as a result, they can be applied directly, without the need for mass-matrix diagonalization. Some advantages of an invariant approach to CP physics are discussed in Ref. 32. Elementary applications of our results include establishing circumstances under which sources of CP violation would be purely nonstandard in any specified model and determining the number of CP-violating phases for any choice of couplings and masses.

For the standard model with n generations, a set of necessary conditions for CP invariance may be found as follows. The $SU(2)_L$ doublets q_{La} and singlets u_{Ra} and d_{Ra}, where the index $a=1,\ldots,n$ labels the generation, are defined only to within unitary transformations. A CP transformation on a field f, where f is q_L, u_R, or d_R, may therefore be represented as $f'=U_fCf^*$, where U_f are $n\times n$ unitary matrices and C is the charge-conjugation matrix. Introducing the $n\times n$ nondiagonal mass matrices $(M^u)_{ab}$, $(M^d)_{ab}$ and requiring invariance of the Lagrangian under these CP transformations yields a set of conditions on the unitary matrices U_f. In terms of Hermitian matrices H_f defined by $H_f=M^fM^{f\dagger}$, where $f=u,d$, these conditions are

$$U_q^\dagger H_f U_q = H_f^* \ . \tag{1}$$

Straightforward manipulations[28] lead to conditions independent of the weak basis:

$$\mathrm{tr}([H_u^p,H_d^q])^r=0 \ . \tag{2}$$

Here, the powers p,q,r are positive integers and r is odd.[33]

These conditions have the same form in the limit of exact $SU(2)\otimes U(1)$ symmetry. This is because the quark mass matrices arise in the standard model from matrices of Yukawa coupling constants when the Higgs field acquires a vacuum expectation value v. These Yukawa matrices have the form M^f/v, so we can take over the entire analysis for the exact symmetry limit with merely the redefinition of H_f in terms of the Yukawa matrices. Note also that the origin of CP violation in this picture is most naturally traced not to a diagonalization of mass matrices but instead to a clash in the CP-transformation properties of the gauge and Yukawa couplings.

The number of independent CP constraints for the case of n generations is readily ascertained from Eq. (1). For simplicity, take H_u nondegenerate. Then, choosing the weak basis in which H_u is diagonal, Eq. (1) with $f=u$ implies that U_q must be a diagonal matrix of n arbitrary phases δ_a, $a=1,\ldots,n$. However, using Eq. (1) with $f=d$ it then follows that the phase of the element $(H_d)_{ab}$

must be $(\delta_a - \delta_b)/2$ modulo π, i.e., all phases of H_d are cyclic and are thus determined by $n-1$ parameters. Since a Hermitian matrix has $n(n-1)/2$ phases in general, there must be $(n-1)(n-2)/2$ constraints. This is then the number of constraints necessary for CP invariance; as expected, it agrees with the usual count of independent KM phases for n generations.

The analysis of the CP properties of the standard model neglects possible unitary transformations on the leptons, gauge bosons, and Higgs boson because these do not lead to additional CP-violating phases. The usual argument is that the introduction of phases leads to no new physics because they can all be absorbed into redefinitions of fields. In the present context, the absence of new CP-violating phases is seen, for example, for the leptons by noting that $H_\nu \equiv 0$, so the content of the lepton equivalent of the condition (2) is null. For the remaining fields, the invariance of the Lagrangian under CP transformations leads to no conditions; i.e., there is no clash between the CP transformations of different terms. Hence, there are no new phases to parametrize CP violation.

In the context of a more general model, however, CP-violating phases might be introduced by scalars, gauge bosons, or Majorana fermions, in addition to Dirac fermions. Let us consider a general gauge theory with matter scalars and spinors, with certain well-defined local and global symmetries. If several fields fall into the same irreducible representations of these symmetries, then they are defined only to within a unitary transformation mixing them. A CP transformation on the scalars H, fermions f, and gauge bosons G_μ can then be represented as

$$H' = U_H H^*, \quad f' = U_f C f^*, \quad G'_\mu = (-1)^{\delta_{0\mu}} U_G G^*_\mu \;. \tag{3}$$

These choices implement the correct CP transformations on the kinetic terms. As before, requiring invariance of the whole Lagrangian under these CP transformations results in conditions on the matrices U. Necessary and sufficient conditions for CP invariance may then in principle be derived.

Here, we are interested in the CP properties of the supersymmetric standard model. This has, as field content, all the fields of the standard model plus an extra Higgs doublet together with superpartners for all fields. The form of the Lagrangian in the limit of exact supersymmetry is then completely specified by the symmetries of the standard model and by supersymmetry. When viewed as the low-energy limit of $N=1$ supergravity models with spontaneously broken supersymmetry, the supersymmetric standard model acquires soft-supersymmetry-breaking operators whose precise form depends on the assumptions made at the unification scale. The $SU(2)\otimes U(1)$ symmetry may be broken in the usual way by giving vacuum expectation values to the neutral components H_1^0, H_2^0 of the Higgs-boson doublets.

We begin by discussing the model in the limit of exact supersymmetry. This limit might be of relevance for CP-violating processes occurring in the early Universe at temperatures above the supersymmetry-breaking scale but below the unification scale. Similarly, we neglect the $SU(2)\otimes U(1)$ breaking in the preliminary analysis. We find that the necessary and sufficient conditions for CP invariance in the supersymmetric standard model in the limit of exact supersymmetry and unbroken $SU(2)\otimes U(1)$ are identical to the conditions (2) for the standard model with unbroken $SU(2)\otimes U(1)$.

The most straightforward way to obtain this result is to perform directly on the Lagrangian CP transformations of the type (3). The resulting conditions for invariance may then be manipulated to yield conditions of the form (2). Because of the large number of independently transforming fields, the details of this calculation are not particularly transparent. We present instead a more intuitive argument.

The unitary matrices that appear in the CP transformation (3) can only mix fields transforming as the same irreducible representation of the local and global symmetries of the Lagrangian. Let us first neglect terms involving superpartners. Then, the only changes in the theory relative to the standard model involve the Higgs sector. There are extra gauge-Higgs couplings due to the existence of the second Higgs multiplet. The Yukawa couplings are modified in that one Higgs multiplet couples to the lepton and singlet down-quark fields while the other couples to the singlet up-quark fields. Finally, the Higgs potential is modified. However, none of these changes introduces new conditions on the CP invariance of the Lagrangian. Essentially, this is because there are only two Higgs doublets, and supersymmetry constrains the singlet quarks of a given charge to couple to only one Higgs doublet. Hence, if the superpartners are absent, the conditions for CP invariance of the model are identical to those of the standard model in the limit of unbroken $SU(2)\otimes U(1)$. However, this conclusion cannot be altered by the addition of superpartners. This is because in the limit of exact supersymmetry the model may be written entirely in terms of superfields. It must therefore be possible to implement any CP transformation on superfields, which means that all components of each superfield must transform by the same unitary matrix. Thus, no new conditions for CP invariance are introduced in the supersymmetric extension of the standard model in the exact-symmetry limit.

This conclusion is unaffected by addition of a Fayet-Iliopoulos term in the action because this is trivially CP invariant. It therefore *is* possible to break supersymmetry without affecting the result. However, in effective low-energy Lagrangians based on $N=1$ supergravity models with spontaneous supersymmetry breaking, the breaking of supersymmetry appears instead through soft operators whose precise form depends on assumptions made about the unified theory. As in the limit of exact supersymmetry, the case of softly broken supersymmetry but unbroken $SU(2)\otimes U(1)$ may have physical relevance for the early Universe. Next, we investigate symmetry breaking of this type. In the interest of generality, we make no restrictions on the soft operators other than requiring that they be compatible with the global and local symmetries of the exact-symmetry limit.

We consider first soft terms involving scalar leptons and scalar quarks. These have the form[34]

$$\mathcal{L} \supset \sum_f (\tilde{\mu}_f^2)_{ab} \tilde{f}_a^* \tilde{f}_b + [(\tilde{\mu}_{Ye})_{ab} \epsilon^{ij} \tilde{l}_a^i H_1^j \tilde{e}_b^{c*} + (\tilde{\mu}_{Yd})_{ab} \epsilon^{ij} \tilde{q}_a^i H_1^j \tilde{d}_b^{c*} + (\tilde{\mu}_{Yu})_{ab} \epsilon^{ij} \tilde{q}_a^i H_2^j \tilde{u}_b^{c*} + \text{H.c.}] \ . \tag{4}$$

Here, the sum over f runs over the SU(2)⊗U(1) representations $\tilde{q},\tilde{u}^c,\tilde{d}^c,\tilde{l},\tilde{e}^c$. The tildes indicate that the fields are scalar partners of the quarks and leptons. The indices a,b label the generations, while i,j are SU(2) vector indices. All terms have mass dimension 4.

To analyze the *CP* properties of the Lagrangian when the terms (4) are added, we allow for *CP* transformations on the scalar quark and scalar lepton fields of the type (3). We neglect any *CP* transformations of the Higgs fields in the following, as it can be shown that their inclusion does not affect the conditions obtained. Denoting the five unitary matrices involved in the *CP* transformations of $\tilde{q},\tilde{u}^c,\tilde{d}^c,\tilde{l},\tilde{e}^c$ by V_f with $f=q,u,d,l,e$, respectively, we find that *CP* invariance of the Lagrangian requires the conditions

$$\begin{aligned} &V_f^\dagger \tilde{\mu}_f^2 V_f = (\tilde{u}_f^2)^T, \quad V_e \tilde{\mu}_{Ye} V_l^\dagger = (\tilde{\mu}_{Ye})^T \ , \\ &V_d \tilde{\mu}_{Yd} V_q^\dagger = (\tilde{\mu}_{Yd})^T, \quad V_u \tilde{\mu}_{Yu} V_q^\dagger = (\tilde{\mu}_{Yu})^T \ . \end{aligned} \tag{5}$$

By straightforward manipulations, a set of necessary conditions may be found for *CP* invariance in the supersymmetric standard model with an arbitrary number of generations.[35] In addition to Eq. (2), these conditions may be expressed as

$$\begin{aligned} &\mathrm{tr}[(\tilde{\mu}_q^2)^p,(H_f)^q]^r = 0 \ , \\ &\mathrm{tr}[(\tilde{\mu}_{Yu}\tilde{\mu}_{Yu}^\dagger)^p,(H_f)^q]^r = 0 \ , \\ &\mathrm{tr}[(\tilde{\mu}_{Yd}\tilde{\mu}_{Yd}^\dagger)^p,(H_f)^q]^r = 0 \ , \\ &\mathrm{tr}[(\tilde{\mu}_{Ye}\tilde{\mu}_{Ye}^\dagger)^p,(\tilde{\mu}_l^2)^q]^r = 0 \ . \end{aligned} \tag{6}$$

Here, H_f is formed from Yukawa matrices for $f=u,d$, while p,q,r are integer and r is odd.

As an elementary application of these results, using methods similar to those for the standard model, we find a total of $4(n-1)(n-2)$ new independent conditions for *CP* invariance of the Lagrangian when the soft terms (4) are added. They arise essentially because the eight matrices $\tilde{\mu}_f^2$, $(\tilde{\mu}_{Ye}\tilde{u}_{Ye}^\dagger)$, $(\tilde{\mu}_{Yd}\tilde{\mu}_{Yd}^\dagger)$, and $(\tilde{\mu}_{Yu}\tilde{\mu}_{Yu}^\dagger)$ are constrained to have cyclic phases. Note that for three generations this shows the existence in the general case of eight new *CP*-violating phases, in addition to the usual KM phase of the standard model. Note also that for the sake of generality we have not assumed any particular form for the matrices $\tilde{\mu}_f^2,\tilde{\mu}_{Ye},\tilde{\mu}_{Yd},\tilde{\mu}_{Yu}$. If some of these matrices are chosen to be real, a smaller number of independent *CP* constraints results. In particular, if *all* soft-supersymmetry-breaking parameters are real, then the number of independent *CP* constraints coincides with that of the standard model. This agrees with a result of Ref. 23.

This completes the analysis for the soft-supersymmetry-breaking terms of Eq. (4). Although other soft terms can also be introduced, they do not affect our results. This is because all remaining possible soft operators are quadratic either in the Higgs fields or in the gaugino fields. The *CP* transformation properties of such terms do not clash with those of other terms in the Lagrangian, so no new conditions are introduced.

We see that various sources of *CP* violation can occur in a general supersymmetric model. In any given model, only a subset of these may be present. The necessary conditions (6) for *CP* invariance are useful in determining this subset. Consider, for example, the interesting class of models for which *CP* violation is solely due to the interaction of the low-energy superpartners, without relation to the usual weak interactions. In this class of models, the KM matrix is real. Therefore, the necessary conditions (2) are satisfied, but some of the conditions (6) are violated. A model of this type has been proposed in Ref. 24.

Let us now briefly discuss the consequences of breaking the SU(2)⊗U(1) symmetry. In a general model with nonminimal Higgs sector, an analysis similar to that above is likely to provide further conditions. However, this is not always the case. Here, we restrict ourselves to showing that no further conditions arise in a simple minimal model with real gaugino and Higgsino mass parameters.

When the two neutral Higgs fields in this model acquire vacuum expectation values, the SU(2)⊗U(1) symmetry breaks to the electromagnetic U(1). Among other effects, this gives masses to the quarks and leptons and modifies the scalar-quark and scalar-lepton masses. These mass changes do not affect the results already obtained, however, because the presence of the Higgs fields as such did not play a role in the analysis. Thus, the existence of vacuum expectation values results only in trivial scalings. Therefore, new conditions for *CP* invariance could only arise from the resulting modifications to the gauge, gaugino, Higgs, or Higgsino sectors of the Lagrangian. However, as might be expected from the standard-model case, those terms involving only gauge and Higgs bosons do not yield new constraints. This leaves only a few terms describing the interactions of gauginos and Higgsinos. These fields comprise two charged Dirac spinors $\tilde{W}$ and $\tilde{H}$ and four Majorana spinors $\tilde{W}^0,\tilde{B}^0,\tilde{H}_1^0,\tilde{H}_2^0$. Since the SU(2)⊗U(1) symmetry is now broken, the charged states can mix, as can the neutral ones. Nonetheless, no new conditions arise.[36] Thus, in this simple model the breaking of SU(2)⊗U(1) has no effect on the analysis.

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